A PROBABILITY FORM OF THE DIFFUSION EQUATION AND ITS USE TO DESCRIBE THE DISTRIBUTION OF THE SOLID PHASE PARTICLE CONCENTRATIONS IN A MECHANICALLY AGITATED CHARGE*

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An attempt is made in this paper to describe the solid particle motion in a flowing (agitated) incompressible liquid on the assumption that this motion may be considered as a diffusion Markov process. It is shown that such a procedure leads to a relation which differs from the diffusion equation commonly used in chemical engineering by the form of diffusion term. The expression proposed, unlike the relation usually used, makes it possible to describe the local concentration extremes of solid phase in the charge mixed.

The description of the solid particle motion in a mechanically agitated charge is generally a very demanding task. Exact relations for expressing the flow of such a two-phase system are too complicated[1,2], and can not be practically solved for such a complex arrangement as, *e.g.*, a vessel equipped with a rotary mechanical impeller, or, if need be, with various internals.

Therefore, usually a subject of interest of chemical-engineering study of such systems is the investigation of their overall characteristics such as the input power of impeller and differently defined so-called critical frequencies of the impeller revolutions which aim at finding the appropriate regime of mixing apparatus from the point of view of suspending the solid particles. The results of research of this type are summarized in monographs on mixing[3-5].

Substantially less attention has hitherto been paid to the local characterization of the solid phase–liquid system, *i.e.*, to the distribution of concentrations in the volume of apparatus.

Mathematic description has consisted in the use of an equation of turbulent diffusion applied even to the two-phase flow motion in other arrangements or to the homogenization of miscible liquids in agitated (incompressible) charge[3]. In a general

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form it is possible to write this equation by the relation

$$\frac{\partial \varrho_s}{\partial t} + (\nabla \varrho_s) \cdot \mathbf{v} - \nabla \cdot (D_s^T \nabla \varrho_s) = 0, \tag{1}$$

where the symbol ϱ_s denotes the solid phase concentration in liquid, $\mathbf{v}$ the velocity of suspension and D_s^T the eddy diffusivity. The latter quantity is, however, in general a function of spatial coordinates, and is usually considered as well as a second-order tensor[6-8]. In the concrete, above-mentioned cases[3-5], however, this quantity was considered as a constant in the whole volume of charge.

In this work, an approach is proposed which leads to an equation analogous to relation (1) with a varying diffusion coefficient, however, in the form which, as it will be shown below, enables one to describe local maxima of the solid phase concentration in charge.

THEORETICAL

Let us consider a part of space limited by impermeable walls with regard to mass transfer which is filled-up with a charge consisting of incompressible liquid and a set of solid particles of component A of the same mass and the same shape. A source of momentum operates in the charge, moving in such a way that the resulting motion of the charge is random.

In the space considered let us define a point which will be chosen as an origin of orthonormed system.

The position of each particle will then be expressed in terms of the position vector $\mathbf{R}_i(t)$ whose end tracks the position of the particle centre of gravity (the subscript i denotes that the i-th particle from a set of N particles is considered). This vector is in general a random function of time; the velocity of the i-th particle is then defined by the relation

$$\mathbf{V}_i(t) \equiv \mathrm{d}\mathbf{R}_i(t)/\mathrm{d}t. \tag{2}$$

On each of the solid particles then act partly external forces, partly interactions with the other particles and with the liquid which surrounds these particles. It would then be possible to write the equation of motion for each the particle in the form of the second Newton law. The resultant force causing the time change in momentum of a given particle would then be — as it follows from statistical mechanics[9] — generally a function of position vectors and velocities of all the particles, the velocity, or even acceleration, of liquid and some other quantities (properties of particles and liquid):

$$m_i \frac{\mathrm{d}\mathbf{V}_i}{\mathrm{d}t} = \mathbf{F}_i[\mathbf{R}_1, ..., \mathbf{R}_N, \mathbf{V}_1, ..., \mathbf{V}_N, \mathbf{V}_B, ...], \quad [i = 1, N]. \tag{3}$$

(Subscript B is here assigned to liquid). The form of the set of these equations could apparently be given more precision, however, in no case solved. Therefore we introduce at his point strongly restricting assumptions which, however, considerably simplify the situation.

But first we define the term needed when formulating the presuppositions on the motion of liquid:

The virtual displacement of liquid will be regarded as the product of the liquid velocity as a random function of time and spatial coordinates and differential time interval, *i.e.*

$$\mathrm{d}\mathbf{Z}(\mathbf{x}, t) = \mathbf{V}_{\mathrm{B}}(\mathbf{x}, t)\, \mathrm{d}t \,. \qquad (4)$$

The expression on the left-hand side can be considered as an infinitesimal displacement of liquid which has occurred at the moment t at the point of space determined by the radius vector $\mathbf{x}$ in the direction of velocity $\mathbf{V}_{\mathrm{B}}$.

Further we shall write down the presuppositions concerning the liquid:

P1. The liquid flow is *a priory* known as a stationary random function of spatial coordinates and time.

P2. The virtual displacement of liquid consists of two parts: deterministic — determined by a mean (expected) value of the liquid velocity and random — conditioned by a fluctuation component of velocity; this random component of the virtual displacement can be expressed as the product of the deterministic function of space coordinates (which can generally be a tensor of the second order) and the differential of the three-dimensional Wiener process[10].

P3. The quantity of particles of component A in charge is so small that it does not significantly influence the motion of the liquid phase.

Now we proceed to the formulation of presuppositions concerning the forces acting on solid particles:

P4. On each particle of component A acts the friction force which is proportional to the difference of velocities of the given particle and the liquid which would occur at the given moment in the centre of inertia of this particle. The coefficient of proportionality is a constant, and the friction force points against the direction of the particle motion.

P5. On each particle acts the mass force which is a deterministic function of the position of centre of inertia of the particle itself.

P6. The resultant of interactions between the particle considered and the other particles of component A and as well between the molecules of liquid and the given particle is a random function of time whose impulse is proportional to the three-dimensional Wiener process. This process and the Wiener process considered in the presupposition P2 are mutually independent. The coefficient of proportionality is generally a scalar function of the position of centre of inertia of the given particle.

In the end we write down a presupposition enabling one to transfer the dynamic problem of description of the particle motion of component A to a kinematic one:

P7. A change of velocity of any particle of component A during a sufficiently long time interval may be neglected with regard to the change of its position in the same time interval.

Discussion of the presuppositions:

The presupposition *sub* P1 expresses only the fact that the idea proposed does not serve for solving the liquid motion in charge under steady-state conditions of mixing; it is considered here as given in advance and completely described. In a general case of random (turbulent) flow it means that the corresponding probability characteristics are known, too. The presupposition P2 further characterizes the random flow of liquid in such a sense that it specifies the character of its velocity fluctuations. It is necessary to note that the presupposition imposes considerable limitations on the time course of these fluctuations for there exists no derivative with respect to time[10] for the Wiener process. The third presupposition is here, in fact, a consequence of the first one; investigation of the effect of component A on the liquid flow would require a form of equation of motion of suspension allowing for the interactions between particles of component A and particles of liquid which influence the own liquid motion. The presupposition P4 postulates a very simple − linear − idea of the relation between the velocity of liquid and those of the particle itself; the particle motion is retarded so that great differences between the velocities cause great changes of the particle momentum in the opposite direction. The point of action of the friction force is hypothetically located into the centre of inertia of particle; this part of the presupposition is to express the idea that on the particle of component A acts above all the liquid which immediately surrounds it. The presupposition P5 specifies the action of external forces on the particle. Finally, the presupposition P6 specifies the effect of particle of component A and of continuous phase (liquid molecules) on the given particle; it characterizes them only as random mutually independent pulses irrespective of the actual values of momentums, or positions of these neighbouring particles. As it will be shown below, the presupposition P7 plays a significant role in deriving the equation of diffusion. Let us note at once, however, that its acceptance is rather problematic. Changes of momentum of particle due to the Wiener process in a short time interval can exhibit a very high value; only if averaged within a sufficiently long time interval with regard to the mentioned retarding action of friction force makes the assumption plausible.

Now let us write the relations following from the presuppositions. The presupposition P1 states that, in a general case, the probability density of a random (three-dimensional) vector field of liquid (see *e.g.*[11]) is given:

$$f_B(\mathbf{v_B}; \mathbf{x}) = f_B(v_{B1}, v_{B2}, v_{B3}; \mathbf{x}) =$$

$$= \frac{\partial^3}{\partial v_{B1} \partial v_{B2} \partial v_{B3}} \, P\{V_{B1}(\mathbf{x}, t) < v_{B1}; \, V_{B2}(\mathbf{x}, t) < v_{B2}; \, V_{B3}(\mathbf{x}, t) < v_{B3}\} \,, \qquad (5)$$

where $V_{Bi}(\mathbf{x}, t)$ are the single components of vector field $\mathbf{V}_B$ at a point $\mathbf{x}$ and time t. The function f_B makes it possible to find all the corresponding characteristics: The first moment (expected value) is, *e.g.*, given by the relation

$$\langle \mathbf{V_B}(\mathbf{x}) \rangle \equiv E[\mathbf{V_B}(\mathbf{x}, t)] = \int \mathbf{v_B} f_B(\mathbf{v_B}; \, \mathbf{x}) \, \mathrm{d}\Omega_{v B} =$$

$$= \sum_{j=1}^{3} \mathbf{e}_j \iiint_{-\infty}^{+\infty} v_{Bi} f_B(v_{B1}, v_{B2}, v_{B3}; \, \mathbf{x}) \, \mathrm{d}v_{B1} \, \mathrm{d}v_{B2} \, \mathrm{d}v_{B3} \,. \qquad (6)$$

The last sum of integrals only explains the way of integration, symbol $\mathbf{e}_j$ denotes the single vectors of base.

The second presupposition makes it possible to write the vector field of velocities as a superposition of the deterministic and random contributions:

$$\mathrm{d}\mathbf{Z}(\mathbf{x}, t) = \mathbf{V_B}(\mathbf{x}, t) \, \mathrm{d}t = \langle \mathbf{v_B}(\mathbf{x}) \rangle \, \mathrm{d}t + \mathbf{V_B^*}(\mathbf{x}, t) \, \mathrm{d}t =$$

$$= \langle \mathbf{v_B'}(\mathbf{x}, t) \rangle \, \mathrm{d}t + \mathbf{h_B}(\mathbf{x}) \,.\, \mathrm{d}\mathbf{W_B}(t) \,. \qquad (7)$$

The symbol $\mathbf{V_B^*}$ denotes the random fluctuations of the velocity field which are further specified as a scalar product of tensor $\mathbf{h_B}$ and differential of the three-dimensional Wiener process $\mathbf{W_B}$.

The third presupposition only states that the functions written in Eqs (5)−(7) are not significantly influenced when adding the component A.

The forces acting on the i-th particle can be written in agreement with the single presuppositions:
The friction force is according to the presupposition P4 given by the relation

$$\mathbf{F}_{ti} = -\beta[\mathbf{V}_i(t) - \mathbf{V_B}(\mathbf{R}_i(t))] \,. \qquad (8)$$

The action of the velocity field of liquid is related to the inertial centre of particle whose position is given by the radius vector $\mathbf{R}_i$, β denotes a scalar constant.

The resultant forces of mass forces acting on the particle can be, according to P5, written by the relation

$$\mathbf{F}_{mi} = m_A \mathbf{a}[\mathbf{R}_i(t)] \,, \qquad (9)$$

where m_A is the mass of arbitrary particle of component A and $\mathbf{a}$ the resultant of intensities of mass forces.

The last — random — force as a resultant of pulses can be expressed as the product of a non-random scalar function of the particle position and a differential of the Wiener process:

$$F_{ni} \, dt = \sigma[R_i(t)] \, dW_A(t). \tag{10}$$

The sum of pulses of these forces is equal to the change of momentum of the i-th particle of component A, i.e.

$$m_A \, dV_i(t) = -\beta[V_i(t) - V_B(R_i(t), t)] \, dt + m_A a[R_i(t)] \, dt + \tag{11}$$
$$+ \sigma[R_i(t)] \, dW_A(t).$$

This relation forms along with relation (2) a system of stochastic differential equations making it possible to describe the position and velocity of component A as random functions of time in moving liquid.

The randomizing effect upon the particle has partly the liquid velocity itself (as far as the flow retains turbulent character), partly the motion of particles surrounding the given particle.

The equation written in this way includes, however, a random function of random argument, $V_B(R_i(t), t)$ and does not describe in this form the velocity $V_i(t)$ as the Markov diffusion process[10]. However, if we insert here from Eq. (7) the actual expression for this function, we obtain the relation

$$m_A \, dV_i(t) = -\beta[V_i(t) - \langle v_B(R_i(t)) \rangle] \, dt + m_A a[R_i(t)] \, dt +$$
$$+ \beta \, h_B[R_i(t)] \cdot dW_B(t) + \sigma[R_i(t)] \, dW_A(t) \tag{12}$$

which already makes it possible to write the corresponding partial differential forward Kolgomorov equation for the six-dimensional transition probability density characterizing the distribution of position and velocity of the i-th particle of component A.

In this place, however, we insert from Eq. (2) into (12):

$$m_A \, dV_i(t) + \beta \, dR_i(t) = \beta \langle v_B[R_i(t)] \rangle \, dt + m_A a[R_i(t)] \, dt +$$
$$+ \beta \, h_B[R_i(t)] \cdot dW_B(t) + \sigma[R_i(t)] \, dW_A(t) \tag{13}$$

and neglect, according to the presupposition P7, the first differential on the left-hand side with regard to the second one. After rearranging we have

$$dR_i(t) = \{\langle v_B[R_i(t)] \rangle + m_A a[R_i(t)]/\beta\} \, dt + h_B[R_i(t)] \cdot dW_B(t) +$$
$$+ \sigma[R_i(t)] \, dW_A(t)/\beta. \tag{14}$$

Now we proceed to write the corresponding Kolgomorov equation. Let us define first the transition (conditional) probability density by the relation

$$f'_{Ri}(\mathbf{x}; t \mid \mathbf{x}_0; \tau) = f'_{Ri}(x_1, x_2, x_3; t \mid x_{01}, x_{02}, x_{03}; \tau) \equiv$$

$$\equiv \frac{\partial^3}{\partial x_1 \partial x_2 \partial x_3} P\{R_{i1}(t) < x_1; R_{i2}(t) < x_2; R_{i3}(t) < x_3 \mid R_i(\tau) =$$

$$= x_{01}; R_{i2}(\tau) = x_{02}; R_{i3}(\tau) = x_{03}\},\tag{15}$$

where the subscripts 1, 2, 3 denote the components of the position vector of the i-th particle.

It is possible to show[10] that for this function holds a partial differential equation whose coefficients are connected with those of stochastic differential equation (14)

$$\frac{\partial f'_{Ri}(\mathbf{x}; t \mid \mathbf{x}_0; \tau)}{\partial t} + \nabla \cdot \{[\langle \mathbf{v}_B(\mathbf{x})\rangle + m_A \mathbf{a}(\mathbf{x})/\beta] f'_{Ri}\} -$$

$$- \tfrac{1}{2} \mathrm{Tr}\{[\nabla^2 \cdot \mathbf{h}_B(\mathbf{x}) \cdot \mathbf{h}_B^T(\mathbf{x})] f'_{Ri}\} - 1/2\beta^2 \Delta\sigma^2(\mathbf{x}) f'_{Ri} = 0.\tag{16}$$

Here $\nabla^2 = \nabla\nabla$ expresses the dyadic product of two differential operators, $\Delta = \nabla \cdot \nabla$ is the Laplace operator, $\mathbf{h}_B^T$ is the second-order tensor whose matrix is a transposed matrix with regard to the matrix of tensor $\mathbf{h}$.

The given equation is to be solved in principle at the given initial and boundary conditions.

Here we shall write only the solution of the Cauchy problem, *i.e.*, we shall assume that the position of the i-th particle at initial moment is given; the initial probability density is then the Dirac function

$$f_{Ri}(\mathbf{x}_0; 0) = \delta_x(\mathbf{x}_0 - \mathbf{x}_i^0).\tag{17}$$

Equation (16) is linear with respect to the solution f'_{Ri}; its solution then will be evidently also the (unconditioned) probability density

$$f_{Ri}(\mathbf{x}; t) = \int_{\Omega_R} f'_{Ri}(\mathbf{x}; t \mid \mathbf{x}_0; 0)\, \delta_x(\mathbf{x}_0 - \mathbf{x}_i^0)\, \mathrm{d}\Omega_R,\tag{18}$$

where the integration is carried out over the whole charge volume considered.

With regard to the presuppositions introduced, above all to the presupposition P6 about random interactions between particles we shall obtain stochastic differential equation (13) or (14) for each the particle of component A, and consequently also Eq. (16) which does not depend on the position or momentum of the other particles.

Therefore we shall obtain N independent equations (16) describing the motions of all the solid particles in charge.

The expression $f_{Ri}(x; t) \Delta\Omega_R(x)$ then denotes the probability that the given particle will occur at a moment t in a small subspace $\Delta\Omega_R(x)$ with its centre at a point determined by the radius vector x. Now this function is being used to express the concentration of solid particles in liquid. Let us denote by symbol $I_i(x, t)$ the random function which takes the value of one in the case that the i-th particle occurs in the subspace $\Delta\Omega_R(x)$ and the value of zero in the opposite case. Then the quantity

$$\tilde{\varrho}_A(x, t) \equiv m_A \frac{\sum\limits_{i=1}^{N} I_i(x, t)}{\Delta\Omega_R(x)} \tag{19}$$

denotes evidently the mass concentration of solid particles in the subspace $\Delta\Omega_R(x)$. This quantity is as well a random function. We shall find its expected value

$$\varrho_A(x, t) = E[\tilde{\varrho}_A(x, t)] = m_A \frac{\sum\limits_{i=1}^{N} E[I_i(x, t)]}{\Delta\Omega_R} = m_A \sum\limits_{i=1}^{N} f_{Ri}(x, t) \tag{20}$$

because the expected value of random quantity I_i is directly equal to its probability (see e.g.[12]). The sum of the probability densities of solid particles is therefore proportional to the expected value of their density. With regard to the above-mentioned considerations, the expected value of $\varrho_A(x, t)$ is also the solution of Eq. (16).

Now we put yet formally

$$V_A^+(x) \equiv m_A a(x)/\beta$$

$$D_B^T(x) \equiv \tfrac{1}{2} h_B(x) \cdot h_B^T(x) \tag{21}$$

$$D_A(x) \equiv \frac{1}{2} \frac{\sigma^2(x)}{\beta^2}$$

and insert into Eq. (16). With regard to the discussion of Eq. (20) we obtain finally

$$\frac{\partial \varrho_A(x, t)}{\partial t} + \nabla \cdot \{\varrho_A(x, t) [\langle v_B(x) \rangle + V_A^+(x)]\} - \mathrm{Tr}\, [\nabla^2 \cdot \varrho_A(x, t)\, D_B^T(x)] -$$
$$- \Delta[\varrho_A(x, t)\, D_A(x)] = 0 . \tag{22}$$

The quantity V_A^+ denotes the velocity which the particle would have in steady state in liquid at rest. In a turbulent medium, this quantity can be considered only as

a correction of the solid phase velocity. This not very satisfactory conclusion results from the use of the "bold" presupposition P7.

The tensor $\mathbf{D}_B^T$ is to be considered as the eddy diffusivity, conditioned, as it follows with regard to Eq. (7), by the square of the liquid velocity fluctuations. The scalar D_A expresses mutual interactions of solid particles.

In case of isotropic turbulence, the eddy diffusivity can be as well considered to be a scalar[8], *i.e.*, $\mathbf{D}_B^T = \mathbf{I}D_B^T$, where $\mathbf{I}$ is the identity tensor. Therefore putting finally

$$\mathbf{V}(\mathbf{x}) = \langle \mathbf{v}_B(\mathbf{x}) \rangle + \mathbf{V}_A^+(\mathbf{x})$$

and

$$D_B^T(x) + D_A(x) = D_A^T(x) \tag{23}$$

and considering that the quantity $\mathbf{V}_A^+(\mathbf{x})$ does not depend significantly on the spatial coordinates, then for an incompressible liquid, when $\nabla . \mathbf{v}_B(\mathbf{x}) = 0$, we obtain the equation

$$\frac{\partial \varrho_A}{\partial t} + \mathbf{V}(\mathbf{x}) . \nabla \varrho_A - \Delta(D_A^T(x) \varrho_A) = 0 \tag{24}$$

which is, with the exception of the position of coefficient D_A^T with respect to the operator of derivative, identical with the commonly used relation (1).

We shall show that just this last form makes it possible to describe in steady state, unlike Eq. (1), the local concentration maxima of the solid phase in charge. In steady state holds evidently the condition $\partial \varrho_A / \partial t = 0$. The necessary condition of the existence of extreme of a function are further the zero first-order derivatives of concentrations, *i.e.*, $\partial \varrho_A / \partial x_1 = \partial \varrho_A / \partial x_2 = \partial \varrho_A / \partial x_3 = 0$.

At a point where the function $\varrho_A(\mathbf{x})$ has an assumed extreme, Eq. (24) reduces to the relation

$$\varrho_A \Delta D_A^T = -D_A^T \Delta \varrho_A, \quad \text{(extreme)}. \tag{25}$$

The left-hand side of the equation is not generally equal to zero; in dependence on its sign, the second-order derivatives of concentrations can form a positive or negative definite matrix, which means that the function exhibits either minimum or maximum.

An analogous rearrangement of Eq. (1) leads to the expression

$$D_s^T \Delta \varrho_s = 0, \quad \text{(extreme?)} \tag{26}$$

from which follows that at a point with zero first-order derivatives, the signs of the second-order derivatives have to necessarily differ — the given expression therefore does not make it possible to describe an extreme. Thus, the aim of experiments

performed was among others to prove the existence of similar extremes in a charge solid phase-liquid agitated with a mechanical rotary impeller.

EXPERIMENTAL

The experiments were carried out in a cylindrical conical-bottomed vessel 0·29 m in diameter with four radial baffles. The vessel was in addition provided with a cylindrical draft-tube (Fig. 1). A six-blade impeller with inclined (45°) plane blades was used which, when rotating, pumped liquid to the bottom. Two ratios of the impeller-to-vessel diameters d/D, *viz.* 0·333 and 0·4 were used. The liquid charge was formed by water and glycerol solution. The solid phase was a graded glass ballotini: the spherical particles were of mean diameter $0·45 . 10^{-3}$ and $0·925 . 10^{-3}$ m and of density 2 640 kg/m^3. The solid phase concentration was equal 4 and 8 volume %. The impeller frequency of revolutions was chosen so as to approach approximately to the critical value for suspension and varied within $4·0 - 7·0$ s^{-1}.

The solid phase concentration was determined by a calibrated capacity sensor[13]. The concentration was measured in a vertical plane between the draft-tube and the vessel wall (hatched surface in Fig. 1) always in 24 places under the same conditions of mixing[14].

Two examples of the experimental results are illustrated in the form of isoconcentration curves in Figs 2 and 3. They are denoted by the letter *a*.

The values of mean liquid velocity as a function of radial and axial coordinate obtained on an equipment with analogous geometrical arrangement[15] and, as well, the data on fluctuations[16] making it possible to calculate the value of eddy diffusivity (however, only as a function of axial coordinate) were moreover available.

RESULTS AND DISCUSSION

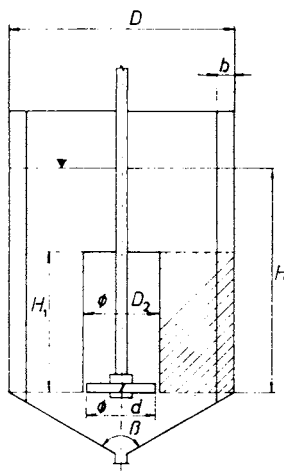To describe the results, Eq. (22) was used which was further simplified on using these assumptions:

Fig. 1

Sketch of experimental equipment. (The hatched region depicts the region of solution of the diffusion equation.) $H_1 = 2/3H$, $b = 0·1D$, $D_2 = 1·1d$, $H = D$, $\beta = 120°$, $d/D = 1/3$ and $2/5$

The spatial distribution of the solid phase in charge is stationary.

The gravitational force plays significant role in the system. (As an attempt, the effect of centrifugal force was also taken into consideration.)

The cylindrical symmetry manifests itself in the system; concentration does not depend on the angular coordinate.

The ratio of eddy diffusivities of solid particle and liquid is equal to the ratio of squares of values of the fluctuation components of velocities of both phases.

The eddy diffusivity of liquid and the last ratio were calculated by means of the relations recommended by Lewins and Glastonburry[17].

The dependence of D_A^T on axial coordinate was approximated by a third-order polynomial; an example of its graphical representation is given in Fig. 4.

Then it was possible to write Eq. (22) in cylindrical coordinates

$$\left(\langle v_{Br}\rangle + v_{Ar}^+\right)\frac{\partial c}{\partial r} + \left(\langle v_{Bz}\rangle + v_{Az}^+\right)\frac{\partial c}{\partial z} - D_A^T\left(\frac{1}{r}\frac{\partial c}{\partial r} + \frac{\partial^2 c}{\partial r^2} + \frac{\partial^2 c}{\partial z^2}\right) -$$

$$- 2\frac{\partial c}{\partial z}\frac{\partial D_A^T}{\partial z} - c\frac{\partial^2 D_A^T}{\partial z^2} = 0 . \tag{27}$$

a b
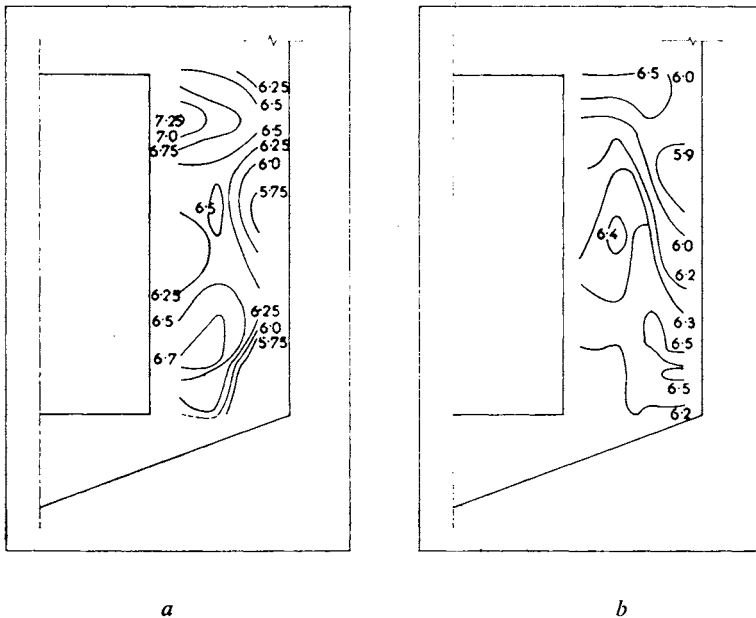
Fig. 2

Isoconcentration curves.
a Measured. (Experimental conditions: liquid phase viscosity $3\cdot55 \cdot 10^{-6}\ m^2\ s^{-1}$, solid sphere diameter $0\cdot45 \cdot 10^{-4}$ m, mean volume concentration of solid phase $0\cdot04$, frequency of impeller revolutions $5\cdot5\ s^{-1}$, $d/D = 2/5$.) b Calculated for the same conditions as in Fig. 2a

Symbol r denotes the radial and z the axial coordinate, c denotes the volume concentration of the solid phase. (The derivatives of the D_A^T factor with respect to the radial coordinate are not given; the experimental dependence of D_A^T on this co-
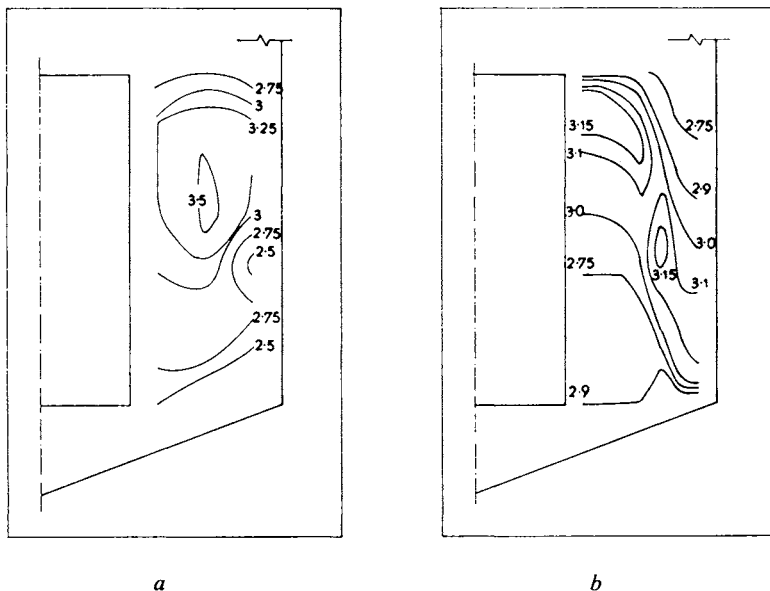

a b

FIG. 3

Isoconcentration curves.
a Measured. (Experimental conditions: frequency of impeller revolutions $3 \cdot 5 \, s^{-1}$, $d/D = 1/3$, otherwise the same as in Fig. 2.) b Calculated for the same conditions as in Fig. $3a$
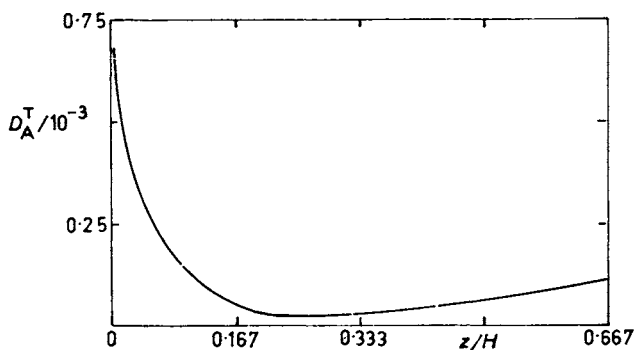

FIG. 4

Dependence of eddy diffusivity of solid particles on axial coordinate for the conditions given in Fig. $2a$

ordinate was not available.) The boundary conditions were determined from the values found experimentally in the upper and lower base of the region investigated (see **Fig. 1**). The boundary conditions on the vertical walls were determined from the condition of their impermeability

$$(\langle v_{\text{Br}}\rangle + v_{\text{Ar}}^{+}) c - D_{\text{A}}^{\text{T}} \frac{\partial c}{\partial r} = 0 . \tag{28}$$

The given set of equations was solved numerically in terms of the finite Gauss––Jordan method[18]. Examples of the calculations are illustrated in the form of isoconcentration curves as well in Figs 2 and 3, and are denoted by the leter b.

As it follows from Figs 2a and 3a under some conditions of mixing, the local extremes of the solid phase concentrations of agitated charge were actually found. The cases in question were above all the glycerol mixtures. From the comparison of the experimental data and the calculated isoconcentration curves (*i.e.* the plots a and b in Figs 2 and 3) it is to conclude that the model proposed exhibits in the best case only a qualitative agreement with experimental results. Let us note, however, that the use of Eq. (*1*) rearranged in the same way as Eq. (*22*), *i.e.* the use of relation (*27*) without two last terms, or even the use of a constant value of eddy diffusivity, did not make it possible in any case to calculate the concentration extremes.

The unsatisfactory agreement between the computation and experiment is caused, regardless of the effect of experimental errors, by applying the kinematic model to the process in which the gravitation, or even the centrifugal force, plays a significant role. The approach presented here aimed at showing where occurs the drawback of applying the equation of diffusion, either in the "probability" or "common" form for describing analogous systems.

A substantially more accurate solution of the problem discussed here could apparently be reached by direct applying relation (*12*) and solving the corresponding Kolgomorov equation for the probability density of the type $f(\mathbf{x}, \mathbf{v}; t)$ where the variable $\mathbf{v}$ describes the solid particle velocity. Such a procedure would make it possible to express in a more real way the effect of forces acting on particles, however, it would lead to far more complicated expressions. Even for a steady-state process with regard to the cylindrical symmetry, it would be necessary to solve partial differential equation of four variables.

The procedure proposed here is therefore from this point of view a compromise; however, it is to draw attention to some other possibilities of using the mathematic apparatus of theory of random processes.

LIST OF SYMBOLS

a	mass force intensity	(m s^{-2})
b	baffle width	(m)

c	volume concentration of solid phase	$(m^3 \, m^{-3})$
d	impeller diameter	(m)
$\mathbf{e}$	unit vector	
f_B	probability density for liquid velocity	$(s^3 \, m^{-3})$
f_R	probability density for particle position	(m^{-3})
$\mathbf{h}_B$	tensor of fluctuations of liquid velocities	$(m \, s^{-1})$
m	mass	(kg)
r	radial coordinate	(m)
t	time	(s)
$\mathbf{v}$	velocity	$(m \, s^{-1})$
$\mathbf{x}$	position vector	(m)
z	axial coordinate	(m)
D	vessel diameter	(m)
D_2	draft-tube diameter	(m)
D^T	eddy diffusivity	$(m^2 \, s^{-1})$
E	operator of expected value	
F	force	$(kg \, m \, s^{-2})$
H	height of liquid in cylindrical part of vessel when impeller at rest	(m)
H_1	draft-tube height	(m)
I	indicator function	
$\mathbf{I}$	identity tensor	
N	number of solid particles	
$\mathbf{R}$	radius vector (random function)	(m)
$\mathbf{V}$	velocity (random function)	$(m \, s^{-1})$
$\mathbf{W}$	Wiener process	$(s^{1/2})$
$\mathbf{Z}$	virtual displacement	(m)
β	friction coefficient	$(kg \, s^{-1})$
δ	Dirac function	
ϱ	mass concentration	$(kg \, m^{-3})$
σ	factor characterizing effect of random force	$(kg \, m \, s^{-3/2})$
Ω_R	volume	(m^3)
τ	time	

Subscripts

i	referring to i-th solid particle
j	referring to j-th coordinate
m	referring to mass force
n	referring to random force
0	referring to initial value
r	referring to radial coordinate
s	referring to solid phase (in "chemical-engineering" expression)
t	referring to friction force
z	referring to axial coordinate
A	referring to solid phase (in "probability" expression)
B	referring to liquid

Superscripts

c	referring to transposed tensor
'	referring to conditioned density of probability
*	referring to fluctuation component
+	referring to the particle in liquid at rest

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